GCE

Further Mathematics B (MEI)

Y415/01: Mechanics B

Advanced Subsidiary GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1	(a)		Use of $T = kx$	B1	3.3	Where k is stiffness	Needs to be used, so we need to see $0.1k$ used for T
			$2k \times 0.1 = 5g$	M1	3.4		Or $0.1x = 2.5g$ (coming from considering the tension in one string).
			$k = 245$ (N m^{-1})	A1	1.1	Accept $25g$	Units not necessary
				[3]			
	(b)		The string may have been stretched beyond its elastic limit	E1	2.2b	OR mass may have had non-elastic collision, oe	OR there is always energy loss in any oscillation
				[1]			OR Air resistance
2	(a)		Must show $3mt = ma$ or $3mt = m\frac{d^2x}{dt^2}$			Using Newton's second law	
			$\frac{d^2x}{dt^2} = 3t$	B1	1.1a	AG	
				[1]			
	(b)		$[x'] = \frac{3}{2}t^2 + A$	M1	1.1	Diff wrt t	Or $x' = \ldots$ found from integration (any constant)
			$\frac{d^2x}{dt^2} = 3t$	A1	1.1	CWO	Or second integration here (consts need not be A/k)
				[2]			Candidate needs to be consistent in whether they are integrating the acceleration or differentiating the given solution

Question			Answer	Marks	AOs		Guidance
	(c)		$6 = \frac{1}{2} + A + k$	B1	3.4	Or $6 = \frac{1}{2} + 10.5 + k$	Might have the value of A already substituted
			$12 = 1.5 + A$	A1	3.1b		
			$A = 10\frac{1}{2}$	A1	1.1		
			$k = -5$	A1	1.1		**B1** can be assumed from the correct value of k
				[4]			
3	(a)		$[a =]\begin{pmatrix}0\\3\end{pmatrix}$	M1	1.2		
			$m\begin{pmatrix}0\\3\end{pmatrix}$	A1	1.1	Accept $3m\mathbf{j}$ oe	**SC** If **M0** then **B1** for 3m (i.e. not as a vector)
				[2]			
	(b)		$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2t + c\\ \frac{3}{2}t^2 - 2t + d\end{pmatrix}$	M1 A1	3.4 1.1	Condone constants omitted for M1A1; allow M1 for integration attempted	$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2t\\ \frac{3}{2}t^2 - 2t\end{pmatrix} + c$
			Attempt to find constants	M1	1.1		Constants might appear as a separate vector.
			$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2t + 2\\ \frac{3}{2}t^2 - 2t - 3\frac{1}{2}\end{pmatrix}$	A1 A1	1.1 1.1		
			Attempt to eliminate t	M1	1.2	Use $t = \frac{1}{2}x - 1$ or their equivalent	
			$y = \frac{3}{8}x^2 - \frac{5}{2}x\ [+0]$	A1	1.1		
				[7]			
	(c)		Particle passes through the origin	B1ft	2.2a	Allow does not pass through origin if their c $\neq 0$	**B0** for "starts at origin"
				[1]			

Question			Answer	Marks	AOs		Guidance
4	(a)		$0.2 \times 0.5 \cos\alpha\ [+0] = 0.2a + 0.5b$	**M1**	**3.3**	CoM Or could be: $0.2 \times 0.5 \times 0.8 = 0.2a + 0.5b$	a and b are speeds of A and B to right parallel to loc after collision
			$b - a = -e(0 - 0.5 \times \cos\alpha)$	**M1**	**3.3**	NEL	**M1** if speed approach / separation reversed
			$0.2a + 0.5b = .08$ and $b - a = 0.3$	**A1**	**1.1**		
			Solve sim equations	**M1**	**1.1**	Get eqn in 1 variable; may be implied by a or b correct.	
			$a = -0.1; b = 0.2$	**A1**	**1.1**		
			Speed of A perp loc = 0.3	**B1**	**1.2**	Allow $0.5\sin\alpha$	
			Speed of A is $\sqrt{(0.3^2 + 0.1^2)}$ Speed of B is $\sqrt{(0.3^2 + 0.2^2)}$	**M1**	**2.2a**	For either	
			Speed of A is 0.316 or $\sqrt{0.1}$ m^{-1} Speed of B is 0.361 or $\sqrt{0.13}$ m^{-1}	**A1**	**1.1**	For both	AEF Allow 2s.f. or better
				[8]			
	(b)		Yes because vel perp loc is same (0.3) for both A and B	**B1**	**2.4**	"speed parallel to the wall" also acceptable	Vertical speed the same is ok for **B1**
				[1]			

Question			Answer	Marks	AOs		Guidance
	(c)		After hitting wall speed of B along loc must be greater than 0.1 m^{-1} towards A	**M1**	**3.1b**		
			Use e = speed of B after / speed of B before collision with wall	**M1**	**3.4**	Setting new horizontal speed equal to $e \times "b"$	
			[e] > 0.5	**A1**	**2.5**	Must have strict inequality	Upper limit of e might also be present. Allow both $0.5 < e < 1$ and $0.5 < e \leq 1$
				[3]			
	(d)		B would not have same speed as A perp loc so after collision with the wall so there would not be a second collision between A and B	**B1**	**3.5a**	No collision is enough for **B1**	
				[1]			
5	**(a)**	**(i)**	Quadratic curve is symmetrical about $x = 2$, and therefore so is L	**E1**	**2.4**	Implied symmetry of lamina about $x = 2$	$x = 2$ is a line of symmetry of the lamina is enough
				[1]			

Question			Answer	Marks	AOs		Guidance
		(ii)	**DR** Area = $\int_0^4(-x^2+4x+2)\mathrm{d}x$	**M1**	**1.1a**	Condone missing limits / dx	
			Area = 18.7 or $18\frac{2}{3}$	**A1**	**1.1**	Or $\frac{56}{3}$	
			$\left(\frac{1}{2}\rho\right)\int(-x^2+4x+2)^2\mathrm{d}x$ $\int x^4-8x^3+12x^2+16x+4\ \mathrm{d}x$	**M1** **A1**	**3.1b** **1.1**	Consider integral of y^2 Correctly expanding y^2 to get x^4 and at least 2 other correct terms.	Condone omission of ρ Condone omission of $\frac{56}{3}\rho\bar{y}=$
			$\left[\frac{1}{5}x^5-2x^4+4x^3+8x^2+4x\right]$	**A1FT**	**1.1**	Follow through on a five term quartic with leading term x^4.	
			$\frac{56}{3}\bar{y}=\frac{232}{5}$	**M1**	**1.1**	Use limits and equate to $\frac{56}{3}\bar{y}$ or $\frac{56}{3}\rho\bar{y}$	
			$[\bar{y}=]$ 2.49 or $\frac{87}{35}$ or $2\frac{17}{35}$	**A1**	**1.1**	2.4857…	
				[7]			
	(b)		$\tan OAG=\frac{2.49}{2}$	**M1**	**3.1b**		
			Angle is 51.2°	**A1**	**1.1**	51.1799…	Allow final answers in range [51.179,51.23]
				[2]			

Question			Answer	Marks	AOs		Guidance
	(c)		The x coordinate will stay the same ($\bar{x} = 2$)	B1	1.1	No reason needed for this one	
			The y coordinate will be less ($\bar{y} < 2.49$) because the part of M not included in N is nearer to the x axis (or less tall).	B1	1.1	"Extra mass is below $y = 2.49$"	Any answer which applies that there is more mass / area below $\bar{y} < 2.49$ than before.
				[2]			
6	(a)		At S, force towards centre = $\frac{m \times 0.64ag}{a}$ or Weight component along SO = $mg\cos 30°$	B1	2.1		Candidates might consider energy first.
			$0.866mg > 0.64mg$ so P does not leave hemisphere	E1	1.1	Or $R = mg\cos 30° - 0.64mg$ $= 0.226mg > 0$	
			Energy at S = $\frac{1}{2}m \times 0.64ag + mga\cos 30°$	M1	2.1		
			This is greater than mga (PE at top) so P passes through H	A1	1.1	Or show that v^2 is positive at H	$v^2 = 0.372ag$
				[4]			
	(b)		$g\sin\theta$	B1	1.2		
				[1]			

Question			Answer	Marks	AOs		Guidance
	(c)		At Q, $mg\cos\theta = \frac{mv^2}{a}$	M1	3.3		
			$\frac{1}{2}m \times 0.64ag + mga\cos 30°$ $= \frac{1}{2}mv^2 + mga\cos\theta$	M1	1.1		
			Eliminate v (or $\cos\theta$)	M1	3.3		Must be in an attempt to balance energy.
			$\cos\theta = \frac{1}{3}(0.64 + 2\cos 30°)$ Or $\frac{1}{3}(0.64 + \sqrt{3})$	A1	1.1	0.79068…; or $\theta = 37.75°$	
				[4]			
	(d)		$v^2 = 7.748664a$	M1	1.1	Or $0.791ag$; or $v = 2.7836\sqrt{a}$	Follow through from part (c) (Method mark)
			$a\cos\theta = 2.7836\sqrt{a}\sin\theta t + \frac{1}{2}gt^2$	M1	3.4	Motion vertically	BOD for initial vertical height of $a\sin\theta$
			$t = 0.2638\sqrt{a}$	A1	1.1	Condone sight of −0.611	
			OL = $a\sin\theta + 2.7836\sqrt{a}\cos\theta \times (0.2638\sqrt{a})$	M1	3.4		
			$1.19a$	A1	1.1	$1.1922a$	If more than 3 s.f shown then allow answers in the range [1.192,1.194]
				[5]			

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